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Self-consistent collective subspaces and diabatic/adiabatic motion in nuclei

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Abstract

We discuss the application of a theory of large-amplitude collective motion to a simple model mimicking the pairing-plus-quadrupole model of nuclear physics.

I. INTRODUCTION

In order to describe processes in nuclei involving large excursions from equilibrium, such as shape-coexistence or fission, we need to go beyond the simple harmonic oscillator picture underlying the random-phase approximation (RPA). One method is to use a method that self-consistently selects collective coordinates for large amplitude collective motion (LACM). Recently we have analysed properties of collective motion in a pairing model Hamiltonian with level crossings [1], where we used a version of a theory proposed by Klein et al [2]. to determine the collective coordinates. We have shown that the system automatically selects either diabatic or adiabatic collective surfaces according to the strength of the pairing interaction. The model used in that paper was rather artificial, since it involved a macroscopic system coupling to a few fermionic degrees of freedom. We felt it desirable to test the theory for a fully microscopic Hamiltonian which is able to describe systems from vibrational nuclei to well-deformed nuclei.

In this paper, we investigate the collective motion in a microscopic model which describes a system of nucleons interacting through a simplified version of the pairing plus quadruple force [3]. Although the Hamiltonian has a very simple form, we shall see that the model can represent several of the interesting phenomena observed in real nuclei.

II. THEORY

The theory of adiabatic large amplitude collective motion (ALACM, see reference [2] for a complete description) is applicable to a classical Hamiltonian system which has kinetic terms only quadratic in momentum. We thus have to start with a truncated Hamiltonian

$$\mathcal{H}(\xi,\pi)=\frac{1}{2}B^{\alpha\beta}\pi_\alpha\pi_\beta+V(\xi)\ , \qquad \alpha,\beta=1,\cdots,n\ , \tag{1}$$

where the mass tensor $B^{\alpha\beta}$, in general, depends on the coordinates ξ^α and is defined by truncation of the Hamiltonian to second order.

Collective coordinates q^i and intrinsic (non-collective) coordinates q^a which are approximately decoupled from each other, are assumed to be obtainable by making a point transformation,

$$q^i = f^i(\xi) \quad (i = 1, \cdots, K), \qquad q^a = f^a(\xi) \quad (a = K+1, \cdots, n) \ . \tag{2}$$

Requiring approximate decoupling between the collective and non-collective coordinates, we find a set of coupled equations. In the case of a single collective coordinate ($K = 1$), the equations can be written as [2]

$$V_{,\alpha} = \lambda f^1_{,\alpha}, \qquad B^{\beta\gamma} V_{;\alpha\gamma} f^1_{,\beta} = \omega^2 f^1_{,\alpha} \ . \tag{3}$$

The equations (3) can be solved iteratively, starting from a stationary point. In principle, the procedure to find a collective path is to find successive points at which an eigenvector $f^1_{,\alpha}$ of the covariant RPA equation (second equation) satisfies the force condition (first equation) at the same time.

III. THE MODEL

The model Hamiltonian we discuss in this paper has $O(4)$ symmetry.It has been originally developed to describe $K^\pi = 0^+$ excitations in deformed nuclei [3]. It has also been used to test various methods for calculation of collective excitations [4–6]. The generalisation to multiple shells has also been formulated, in order to investigate shape-coexistence phenomena [7].

Consider a single shell with angular momentum j, and define the four basic operators,

$$P^\dagger = \sum_{m>0} c^\dagger_m c^\dagger_{\bar{m}} \ , \quad \tilde{P}^\dagger = \sum_{m>0} \sigma_m c^\dagger_m c^\dagger_{\bar{m}} \ , \tag{4}$$

$$N = \sum_m c^\dagger_m c_m \ , \quad Q = \sum_m \sigma_m c^\dagger_m c_m \ . \tag{5}$$

Here $\sigma_m = -1$ if $|m| < \Omega/2$, and $+1$ for the other values of m, which mimics properties of the matrix elements of the quadrupole operator $r^2 Y^2_0$. $\Omega = j + 1/2$ is assumed to be an even integer. Hereafter we call Q the quadrupole operator. The four operators, P, $\tilde{P}$, N and Q, close under commutation, and generate the Lie algebra $so(4)$.

We now build a model Hamiltonian with $O(4)$ dynamical symmetry

$$H = -G P^\dagger P - \frac{1}{2}\chi Q^2 \ , \tag{6}$$

which contains both a monopole pairing and a quadrupole-quadrupole (QQ) interaction. The exact solution can be obtained by diagonalisation in an $O(4)$ basis.

The behaviour of the model is based in the competition between the pairing force which tends to align quasi-spins and the QQ force which tends to de-align them. This can be made extremely clear in the limit of no pairing or no QQ force, where the model can be solved exactly.

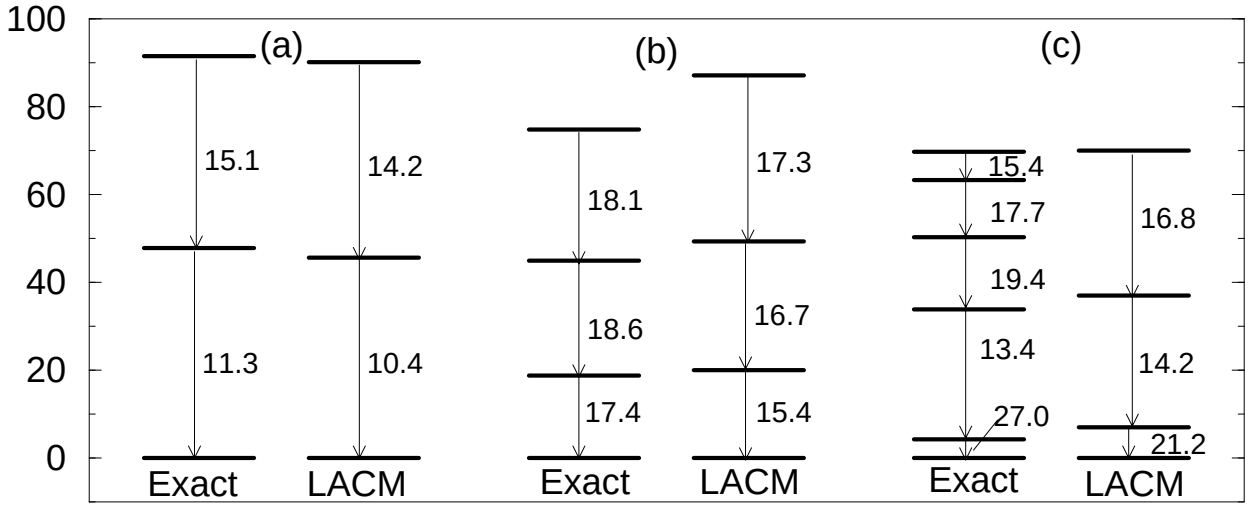

FIG. 1. Excitation energies and transition matrix elements $|\langle n|Q|n-1\rangle|$ (numbers next to arrows) in the two-shell $O(4)$ model discussed in the text, as functions of quadrupole force strength χ. The case (a) represents the almost harmonic motion for $\chi = 0.1$, (b) the flat potential for $\chi = 0.3$ and (c) shape coexistence for $\chi = 0.4$. In each case we compare an exact diagonalisation with the result of requantisation in a single collective variable.

We can easily generalise the previous model into a multi-j-shell case, $(j_1, j_2, \cdots, j_\Lambda)$. For each j-shell, we take $\Omega_i = j_i + 1/2$ to be even, and we weigh the quadrupole operators by q_j, the magnitude of quadrupole moment carried by the single-particle state j. We again use the pairing plus quadrupole Hamiltonian, but we now add spherical single-particle energies,

$$H = \sum_{jm} \epsilon_j c^\dagger_{jm} c_{jm} - GP^\dagger P - \frac{1}{2}\chi Q^2 \ . \tag{7}$$

IV. RESULTS AND CONCLUSION

One of the representative calculations we performed is for a two-shell problem. Both shells have $\Omega = 10$, but we take $q_1 = 3, q_2 = 1$ and $\epsilon_1 = 0, \epsilon_2 = 10$. For strong "deformation" we expect that the shells mix strongly, and for weak QQ force we expect a simple spectrum. In this case, as can be seen in figure 1, we find that as we move from a harmonic spectrum to a more complex mixed situation, that we still keep reasonable agreement for both spectrum and transition strengths. The values of the QQ strength used were such that the middle case exactly captures the turn-over from single to double well. In all these cases one collective coordinate appears to be enough. There are some states in the double-well structure that are not well described by the current method, which is currently under investigation.

In conclusion we have shown that our method can deal with both anharmonic vibrations and shape coexistence. In future we shall extend this research to the proton-neutron O(4) model, and to the real pairing-plus-quadrupole model.

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